2.4 Solving Multi-Step Inequalities

Essential Question

How can you solve a multi-step inequality?

EXPLORATION 1 Solving a Multi-Step Inequality

Work with a partner.

- Use what you already know about solving equations and inequalities to solve each multi-step inequality. Justify each step.
- Match each inequality with its graph. Use a graphing calculator to check your answer.

JUSTIFYING STEPS

To be proficient in math, you need to justify each step in a solution and communicate your justification to others.

a. $2x + 3 \leq x + 5$

b. $-2x + 3 > x + 9$

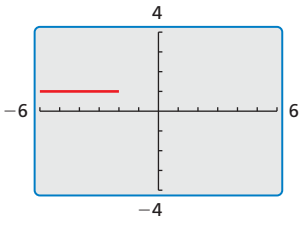
c. $27 \geq 5x + 4x$

d. $-8x + 2x - 16 < -5x + 7x$

e. $3(x - 3) - 5x > -3x - 6$

f. $-5x - 6x \leq 8 - 8x - x$

A.



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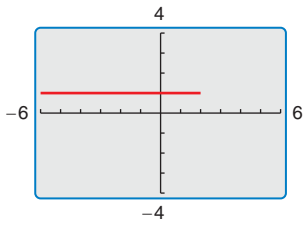

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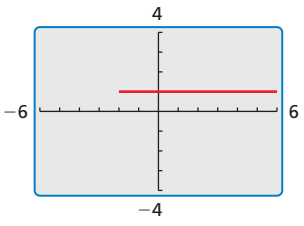

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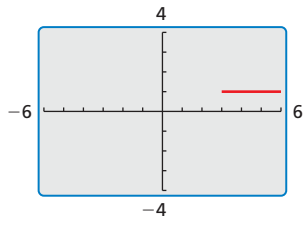

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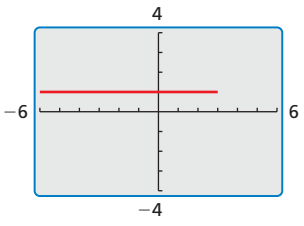
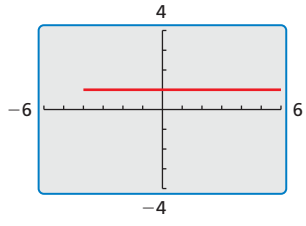


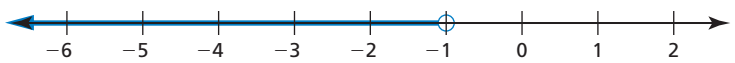
F.



Communicate Your Answer

2. How can you solve a multi-step inequality?

3. Write two different multi-step inequalities whose solutions are represented by the graph.



2.4 Lesson

What You Will Learn

- Solve multi-step inequalities.
- Use multi-step inequalities to solve real-life problems.

Solving Multi-Step Inequalities

To solve a multi-step inequality, simplify each side of the inequality, if necessary. Then use inverse operations to isolate the variable. Be sure to reverse the inequality symbol when multiplying or dividing by a negative number.

EXAMPLE 1 Solving Multi-Step Inequalities

Solve each inequality. Graph each solution.

a. $\frac{y}{-6} + 7 < 9$ **b.** $2v - 4 \geq 8$

SOLUTION

a. $\frac{y}{-6} + 7 < 9$ Write the inequality.

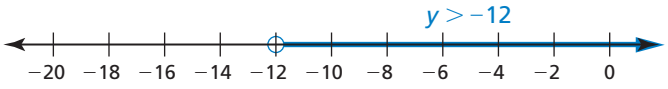
$-7 \quad -7$ Subtract 7 from each side.

$\frac{y}{-6} < 2$ Simplify.

$-6 \cdot \frac{y}{-6} > -6 \cdot 2$ Multiply each side by –6. Reverse the inequality symbol.

$y > -12$ Simplify.

The solution is $y > -12$.



b. $2v - 4 \geq 8$ Write the inequality.

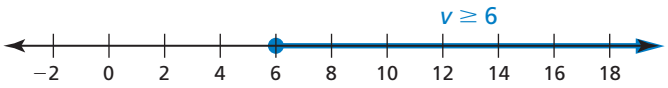
$+4 \quad +4$ Add 4 to each side.

$2v \geq 12$ Simplify.

$\frac{2v}{2} \geq \frac{12}{2}$ Divide each side by 2.

$v \geq 6$ Simplify.

The solution is $v \geq 6$.



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Solve the inequality. Graph the solution.

1. $4b - 1 < 7$ **2.** $8 - 9c \geq -28$

3. $\frac{n}{-2} + 11 > 12$ **4.** $6 \geq 5 - \frac{v}{3}$

EXAMPLE 2 Solving an Inequality with Variables on Both Sides

Solve $6x - 5 < 2x + 11$.

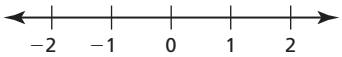
SOLUTION

$6x - 5 < 2x + 11$	Write the inequality.
$+5 \quad +5$	Add 5 to each side.
$6x < 2x + 16$	Simplify.
$-2x \quad -2x$	Subtract $2x$ from each side.
$4x < 16$	Simplify.
$\frac{4x}{4} < \frac{16}{4}$	Divide each by 4.
$x < 4$	Simplify.

▶ The solution is $x < 4$.

When solving an inequality, if you obtain an equivalent inequality that is true, such as $-5 < 0$, the solutions of the inequality are *all real numbers*. If you obtain an equivalent inequality that is false, such as $3 \leq -2$, the inequality has *no solution*.

Graph of an inequality whose solutions are all real numbers



Graph of an inequality that has no solution

EXAMPLE 3 Inequalities with Special Solutions

Solve (a) $8b - 3 > 4(2b + 3)$ and (b) $2(5w - 1) \leq 7 + 10w$.

SOLUTION

a.

$8b - 3 > 4(2b + 3)$	Write the inequality.
$8b - 3 > 8b + 12$	Distributive Property
$-8b \quad -8b$	Subtract $8b$ from each side.
$-3 > 12$ ✗	Simplify.

▶ The inequality $-3 > 12$ is false. So, there is no solution.

b.

$2(5w - 1) \leq 7 + 10w$	Write the inequality.
$10w - 2 \leq 7 + 10w$	Distributive Property
$-10w \quad -10w$	Subtract $10w$ from each side.
$-2 \leq 7$	Simplify.

▶ The inequality $-2 \leq 7$ is true. So, all real numbers are solutions.

LOOKING FOR STRUCTURE

When the variable terms on each side of an inequality are the same, the constant terms will determine whether the inequality is true or false.

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Solve the inequality.

5. $5x - 12 \leq 3x - 4$

6. $2(k - 5) < 2k + 5$

7. $-4(3n - 1) > -12n + 5.2$

8. $3(2a - 1) \geq 10a - 11$

Solving Real-Life Problems

EXAMPLE 4 **Modeling with Mathematics**

You need a mean score of at least 90 points to advance to the next round of the touch-screen trivia game. What scores in the fifth game will allow you to advance?

SOLUTION

1. Understand the Problem You know the scores of your first four games. You are asked to find the scores in the fifth game that will allow you to advance.

2. Make a Plan Use the definition of the mean of a set of numbers to write an inequality. Then solve the inequality and answer the question.

3. Solve the Problem Let x be your score in the fifth game.

$$\frac{95 + 91 + 77 + 89 + x}{5} \geq 90$$ Write an inequality.

$$\frac{352 + x}{5} \geq 90$$ Simplify.

$$5 \cdot \frac{352 + x}{5} \geq 5 \cdot 90$$ Multiply each side by 5.

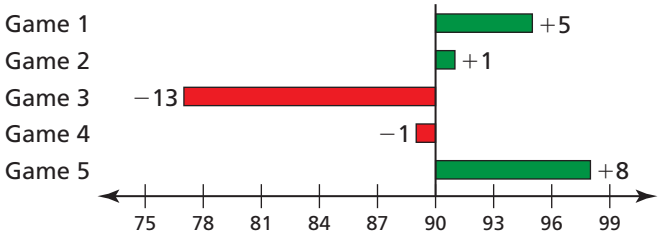
$$352 + x \geq 450$$ Simplify.

$$-352 \quad -352$$ Subtract 352 from each side.

$$x \geq 98$$ Simplify.

▶ A score of at least 98 points will allow you to advance.

4. Look Back You can draw a diagram to check that your answer is reasonable. The horizontal bar graph shows the differences between the game scores and the desired mean of 90.



To have a mean of 90, the sum of the differences must be zero.

$$5 + 1 - 13 - 1 + 8 = 0$$ ✓

REMEMBER

The mean in Example 4 is equal to the sum of the game scores divided by the number of games.

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9. WHAT IF? You need a mean score of at least 85 points to advance to the next round. What scores in the fifth game will allow you to advance?

2.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

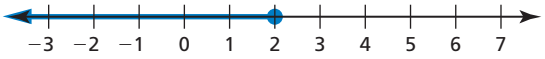
1. **WRITING** Compare solving multi-step inequalities and solving multi-step equations.
2. **WRITING** Without solving, how can you tell that the inequality $4x + 8 \leq 4x - 3$ has no solution?

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, match the inequality with its graph.

3. $7b - 4 \leq 10$
4. $4p + 4 \geq 12$
5. $-6g + 2 \geq 20$
6. $3(2 - f) \leq 15$

A.


B.


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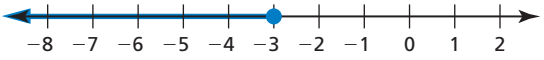

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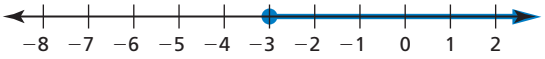


In Exercises 7–16, solve the inequality. Graph the solution. *(See Example 1.)*

7. $2x - 3 > 7$
8. $5y + 9 \leq 4$
9. $-9 \leq 7 - 8v$
10. $2 > -3t - 10$
11. $\frac{w}{2} + 4 > 5$
12. $1 + \frac{m}{3} \leq 6$
13. $\frac{p}{-8} + 9 > 13$
14. $3 + \frac{r}{-4} \leq 6$
15. $6 \geq -6(a + 2)$
16. $18 \leq 3(b - 4)$

In Exercises 17–28, solve the inequality. *(See Examples 2 and 3.)*

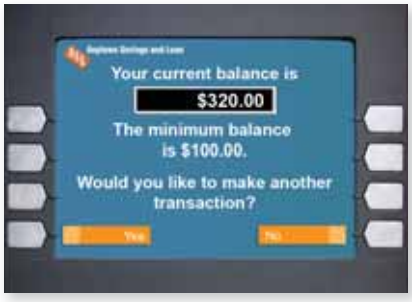
17. $4 - 2m > 7 - 3m$
18. $8n + 2 \leq 8n - 9$
19. $-2d - 2 < 3d + 8$
20. $8 + 10f > 14 - 2f$
21. $8g - 5g - 4 \leq -3 + 3g$
22. $3w - 5 > 2w + w - 7$
23. $6(\ell + 3) < 3(2\ell + 6)$
24. $2(5c - 7) \geq 10(c - 3)$
25. $4\left(\frac{1}{2}t - 2\right) > 2(t - 3)$
26. $15\left(\frac{1}{3}b + 3\right) \leq 6(b + 9)$
27. $9j - 6 + 6j \geq 3(5j - 2)$
28. $6h - 6 + 2h < 2(4h - 3)$

ERROR ANALYSIS **In Exercises 29 and 30, describe and correct the error in solving the inequality.**

29.
$$\frac{x}{4} + 6 \geq 3$$
$$x + 6 \geq 12$$
$$x \geq 6$$

30.
$$-2(1 - x) \leq 2x - 7$$
$$-2 + 2x \leq 2x - 7$$
$$-2 \leq -7$$
All real numbers are solutions.

31. **MODELING WITH MATHEMATICS** Write and solve an inequality that represents how many \$20 bills you can withdraw from the account without going below the minimum balance. *(See Example 4.)*

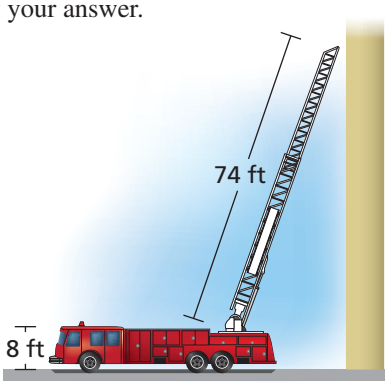


32. MODELING WITH MATHEMATICS A woodworker wants to earn at least $25 an hour making and selling cabinets. He pays $125 for materials. Write and solve an inequality that represents how many hours the woodworker can spend building the cabinet.

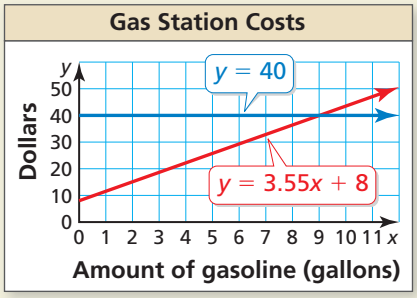
33. MATHEMATICAL CONNECTIONS The area of the rectangle is greater than 60 square feet. Write and solve an inequality to find the possible values of x.

34. MAKING AN ARGUMENT Forest Park Campgrounds charges a $100 membership fee plus $35 per night. Woodland Campgrounds charges a $20 membership fee plus $55 per night. Your friend says that if you plan to camp for four or more nights, then you should choose Woodland Campgrounds. Is your friend correct? Explain.

35. PROBLEM SOLVING The height of one story of a building is about 10 feet. The bottom of the ladder on the fire truck must be at least 24 feet away from the building. How many stories can the ladder reach? Justify your answer.



36. HOW DO YOU SEE IT? The graph shows your budget and the total cost of x gallons of gasoline and a car wash. You want to determine the possible amounts (in gallons) of gasoline you can buy within your budget.



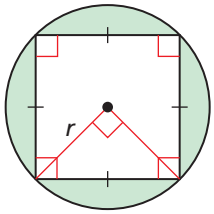
a. What is your budget?

b. How much does a gallon of gasoline cost? How much does a car wash cost?

c. Write an inequality that represents the possible amounts of gasoline you can buy.

d. Use the graph to estimate the solution of your inequality in part (c).

37. PROBLEM SOLVING For what values of r will the area of the shaded region be greater than or equal to $9(\pi - 2)$?



38. THOUGHT PROVOKING A runner's times (in minutes) in the four races he has completed are 25.5, 24.3, 24.8, and 23.5. The runner plans to run at least one more race and wants to have an average time less than 24 minutes. Write and solve an inequality to show how the runner can achieve his goal.

REASONING In Exercises 39 and 40, find the value of a for which the solution of the inequality is all real numbers.

39. $a(x + 3) < 5x + 15 - x$

40. $3x + 8 + 2ax \geq 3ax - 4a$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Write the sentence as an inequality. *(Section 2.1)*

41. Six times a number y is less than or equal to 10.

42. A number p plus 7 is greater than 24.

43. The quotient of a number r and 7 is no more than 18.